Closed-ended contingent valuation surveys are used to assess demands in hypothetical markets and recently have been applied widely to the valuation of (non-market) environmental resources. This interviewing strategy holds considerable promise for more general market research applications. The authors describe a new maximum likelihood estimation technique for use with these special data. Unlike previously used methods, the estimated models are as easy to interpret as ordinary least squares regression results and the results can be approximated accurately by packaged probit estimation routines.

Estimating Willingness to Pay from Survey Data: An Alternative Pre-Test-Market Evaluation Procedure

Market planners and product developers frequently need to assess the market potential of a product that is not yet available for actual test marketing. Lilien and Kotler (1983, p. 330) mention buyer intentions surveys as one way of eliciting potential demand for a product, especially major consumer durable goods. Preference regression models also are used with pre-test-market survey data (Lilien and Kotler, p. 365). Urban and Hauser (1980) evaluated several preference analysis methods used in new product design and concluded that preference regression methods “excel at identifying key physical and psychological benefits” associated with a new product. However, recent developments in techniques for assigning a social value to nonmarket environmental resources also seem ideally suited to market research tasks.

These methods, known as “contingent valuation” (CV) techniques, are a potentially valuable supplement to other pre-test-market methods. Like the Silk and Urban (1978) ASSESSOR model, these survey techniques have the potential to permit low cost screening of different marketing mix elements such as package size, color options, or price. This last dimension of the product deserves more attention than it receives in most models for new products. Lilien and Kotler (p. 736–7) reviewed 14 different test- and pre-test-market models and determined that very few of them attempt to capture price sensitivity, either at the trial stage or at the repeat purchase stage. This oversight in so many models is perplexing; economic theory suggests strongly that the price sensitivity of demand for a particular product is crucial to production and marketing decisions. Contingent valuation models are ideally suited to eliciting willingness to pay and therefore warrant careful consideration.

In closed-ended contingent valuation surveys (or “referendum” surveys, as they are sometimes called in the environmental literature), the attributes of the environmental resource first are established, then the respondent is asked whether or not he or she would pay or accept a single specific sum for access to the resource. The arbitrary sum is varied across respondents. This approach generates a scenario similar to that encountered by con-

Two other approaches to asking the valuation questions are open-ended (the respondent is simply asked to name the sum) and sequential bids (respondents are asked whether or not they would pay or accept some specified sum, then the question is repeated with a higher or lower amount, depending on the initial response, and so on until the true value is finally bracketed).
consumers in their usual market transactions. As a hypothetical price is stated, the respondent merely decides whether to "take it or leave it" and is not required to suggest a specific dollar value.

Comprehensive assessments of the CV approach in an environmental context are provided by Cummings, Brookshire, and Schulze (1986) and Mitchell and Carson (1987). One of the papers included in the former volume (Randall 1986) anticipates the marketing application we propose here. In defense of the contingent valuation technique, Randall points out that

... the goods offered in contingent markets are not always familiar, and individuals may not associate these particular goods with trading possibilities. Nevertheless, unfamiliar goods are often introduced in "real" markets and, especially, in market experiments. So the distinction between "real" and contingent markets is, if anything, a matter of degree.

If the attributes of the proposed product can be described reasonably accurately, contingent valuation survey techniques can provide valuable information about the nature of demand. In a marketing CV experiment the investigator would present potential buyers with a detailed "catalog"-type description of the product, including its price, and ask whether or not they would make a purchase if it were actually available. If the response is "yes," the investigator can infer that the buyer would be willing to pay at least the listed price for this product.

In the following sections, we first describe our new maximum likelihood estimation technique for CV data. We then entertain some of the considerations that must be addressed in formulating empirical specifications for the "valuation" (willingness to pay or inverse demand) function. The final section illustrates the results generated by our method in contrast to those produced by methods used previously for closed-ended CV surveys.

AN INNOVATION IN ESTIMATING MODELS USING CV DATA

Previous empirical analyses of closed-ended contingent valuation data from field experiments employed estimation methods borrowed directly from the tradition of binary choice models for individual response data in the literature on bioassay and from discrete choice models in econometrics. These models seek to predict whether or not the respondent is willing to pay an arbitrarily assigned threshold amount, using the level of the threshold as an explanatory variable. However (as is always the case with these conventional discrete choice models), these estimation methods only allow recovery of estimated choice probabilities as a function of the threshold levels.

Because only probabilities are estimated, we cannot in these models estimate directly how much an individual will be willing to pay. However, average willingness to pay for the whole sample (at the means of the data) can be approximated by a process akin to computing \( \sum p_i t_i \) where \( p_i \) is the estimated probability that each consumer will pay the threshold amount \( t_i \). The inclusion of additional explanatory variables allows more accurate estimation of the probabilities used in the computation of the overall mean valuation. Though rough point estimates describing the apparent sensitivity of willingness to pay to quantities demanded (and other variables) can be obtained, the difficulty of assigning accurate standard errors precludes any form of statistical inference about the estimates. It would be particularly desirable to be able to reconstruct the valuation function directly, but the models are ill-suited to this task.

These shortcomings are serious because important marketing decisions may depend on the extent to which certain attributes of a proposed product contribute to its overall value to consumers. The value to prospective buyers is certainly derived from the levels of—and interactions among—a wide variety of product characteristics (size, weight, color, fragrance, etc.) as well as the characteristics of the consumers themselves (i.e., income, age, sociodemographic group, and individual tastes), and probably even the local consumption environment (unemployment rates, interest rates, consumer optimism, or other general barometers of the health of the economy). Therefore it is essential to know the marginal contributions to “unit” value of all these factors in order to predict the impact on potential sales of (1) minor adjustments to the attributes of the product itself, (2) changes in the composition of the target consumer group, or (3) uncontrollable changes in the local macroeconomic environment. The models described in the following paragraphs can readily accommodate these effects.

Assume that the unobserved continuous dependent variable is the respondent’s true valuation or willingness to pay (WTP) for the product, \( Y_i \). If we can assume that the underlying distribution of \( Y_i \), conditional on a vector of explanatory variables, \( x_i \), has some known distribution with a mean of \( x_i \beta \), maximum likelihood techniques are appropriate.

In the standard binary probit model, we would assume that

\[
Y_i = x_i \beta + u_i
\]

where valuation (willingness to pay) \( Y_i \) is unobserved, but depends on a vector of explanatory variables, \( x_i \) (consisting of elements \( x_{ij}, j = 1, \ldots, p \)). \( Y_i \) is manifested through the discrete indicator variable, \( I_i \), such that

\[
I_i = \begin{cases} 
1 & \text{if } Y_i > 0, \\
0 & \text{otherwise}.
\end{cases}
\]

\[1\]

If the product already exists, a variety of other techniques can be used for evaluating demand, but the method proposed here may be a useful supplement even in those situations.

\[2\]

If we then assume that $u_i$ is distributed $N(0, \sigma^2)$, \(^4\)

\[
\Pr(I_i = 1) = \Pr(Y_i > 0) = \Pr(u_i > -x_i'\beta) = \Pr(u_i/\sigma > -(x_i/\beta)/\sigma) = 1 - \Phi(-(x_i/\gamma))
\]

where $\Phi$ signifies the standard normal cumulative density function and $\gamma = \beta/\sigma$. The log-likelihood function is therefore

\[
\log L = \sum \{ I_i \log [1 - \Phi(-x_i'\gamma)] + (1 - I_i) \log [\Phi(-x_i'\gamma)]\}
\]

and it is not possible to estimate $\beta$ separately because it always appears as $\beta/\sigma$. The model therefore must be evaluated in terms of its estimated probabilities because the underlying valuation function, $x_i'\beta$, cannot be recovered.

With closed-ended contingent valuation survey data, however, each individual is confronted with a threshold value, $t_i$, and by his or her (yes/no) response we conclude that his or her true WTP is either greater than or less than $t_i$. Therefore, the conventional probit model should be modified. As before, we can assume that:

\[
Y_i = x_i'\beta + u_i
\]

with $u_i$ again distributed normally, but we now can employ the threshold value as follows.

\[
I_i = 1 \text{ if } y_i > t_i = 0 \text{ otherwise}
\]

so that

\[
\Pr(I_i = 1) = \Pr(Y_i > t_i) = \Pr(u_i > t_i - x_i'\beta) = \Pr(u_i/\sigma > (t_i - x_i'\beta)/\sigma) = 1 - \Phi((t_i - x_i'\beta)/\sigma)
\]

The log-likelihood function becomes

\[
\log L = \sum \{ I_i \log [1 - \Phi ((t_i - x_i'\beta)/\sigma)] + (1 - I_i) \log [\Phi ((t_i - x_i'\beta)/\sigma)]\}.
\]

The presence of $t_i$ allows $\sigma$ to be identified, which enables us to isolate $\beta$ so that the underlying valuation function can be determined. (Note that if $t_i = 0$, we again have the conventional probit specification.)

The likelihood function in equation 8 can be optimized directly by using a general nonlinear function optimization computer program. \(^5\) This procedure will yield separate estimates of $\beta$ and $\sigma$ and their individual asymptotic standard errors. (Hypothesis testing can be accomplished as usual, using the corresponding asymptotic $t$-test statistics or likelihood ratio test statistics.) However, it is intriguing to note that estimates of $-1/\sigma$ and $\beta/\sigma$ can be obtained from conventional packaged probit algorithms. If we simply include the threshold, $t_i$, among the "explanatory" variables in an ordinary (maximum likelihood) probit model

\[
-(t_i, x') \begin{bmatrix} -1/\sigma \\ \beta/\sigma \end{bmatrix} = -x'*\gamma^*,
\]

the augmented vectors of variables, $x^*$, and coefficients, $\gamma^*$, may be treated as one would treat the explanatory variables and coefficients in an ordinary probit estimation. From $\gamma^*$, it is possible to compute point estimates of the desired parameters $\beta$ and $\sigma$. If we distinguish the elements of $\gamma^*$ as $(\hat{\alpha}, \hat{\gamma}) = (-1/\hat{\sigma}, \hat{\beta}/\hat{\sigma})$, then $\hat{\sigma} = -1/\hat{\alpha}$ and $\hat{\beta} = -\hat{\gamma}/\hat{\alpha}$.

This is good news for practitioners who may not have access to a general function-optimizing computer program. Several statistical packages now offer "probit" commands. However, a little extra effort is required to obtain asymptotic standard errors for the desired $\hat{\alpha}$ and $\hat{\beta}$ parameters because they are functions of the estimated probit parameters. One alternative is to use Taylor series approximation formulas for their variances (see Kmenta 1971, p. 444, or, for an example in the marketing research literature, see Schmittlein and Mahajan 1982).

\[
\text{Var}(\hat{\sigma}) = \text{Var}(-1/\hat{\sigma}) = \frac{1}{\hat{\sigma}^2} \text{Var}(\hat{\alpha})
\]

\[
\text{Var}(\hat{\beta}) = \frac{1}{\hat{\sigma}^2} \text{Var}(\hat{\alpha}) + [-1/\hat{\sigma}^2] \text{Var}(\hat{\gamma}) + 2 \frac{\gamma}{\hat{\alpha}^2}[-1/\hat{\sigma}] \text{Cov}(\hat{\alpha}, \hat{\gamma})
\]

Another way to obtain standard error estimates (if a general function-optimizing program is unavailable) is to use the analytical formulas for the Hessian matrix corresponding to the likelihood function in equation 8 with the optimal values of $\beta$ and $\hat{\sigma}$ derived from $\hat{\gamma}$. The negative of the inverse of this matrix can be used to approximate the Cramer-Rao lower bound for the variance-covariance matrix for $\hat{\sigma}$ and $\hat{\beta}$. Alternatively, the expected values of the Hessian matrix elements are sometimes used in this process. \(^6\) Formulas for the elements of the Hessian matrix and their expectations are given in the Appendix.

Whichever way the asymptotic standard errors are determined for the parameter estimates when they are derived from a conventional probit algorithm (rather than directly), they are useful for hypothesis testing about the signs and sizes of the individual $\hat{\beta}_j$ parameters, an important objective of the modeling process.

\(^4\)A generalized logistic distribution likewise could be assumed and the following exposition could be cast as an analog to the logit model (see Cameron 1987).

\(^5\)We use a package called GQOPT, beginning the optimization with the DFP (Davidon-Fletcher-Powell) algorithm and completing it with the GRADX quadratic hill-climbing method.

\(^6\)Alternatively, the outer product of the gradient vector evaluated at the optimum sometimes is used. However, as the expectation of the Hessian has simple formulas, it probably is preferred in this application.
CANDIDATE SPECIFICATIONS FOR WILLINGNESS-TO-PAY MODELS

In this section it is useful to distinguish several types of variables: \( q_i \) the number of units of the product being valued, and \( x_i = (c_i, z_i, w_i) \), where the \( c_i \) are characteristics of the product being valued, the \( z_i \) are personal characteristics of the individual being asked to make the valuation, and the \( w_i \) are variables describing the current (local) macroeconomic environment (which will include the prices of other goods).

In formulating a model to address willingness to pay, the analyst must keep in mind whether the product being considered is purchased just once or repeatedly.

Valuing a Single Marginal Unit of a Product

In this case, technically we would be considering the inverse demand function for a single unit of a product with a particular configuration of characteristics (such as a large consumer durable good). In practice most analysts will be constrained by deficient data to working with plausible ad hoc specifications for the WTP function. In this context, reasonable first-generation models might include variants of linear \( \log(Y_i) = \beta_0 + \beta_1 q_i + \varepsilon \) or \( \log(Y_i) = \beta_0 + \beta_2 x_i + \varepsilon \) forms. (Whatever transformation is desired for \( Y_i \) is simply imposed on \( T_i \).)

Valuing Total Units of a Product

Alternatively, the proposed product might be a close substitute for a product that the consumer currently purchases regularly (such as public transportation, beer, long distance telephone services, or weekly newsmagazines).\textsuperscript{7} The survey should establish the number of units of the good typically purchased per unit of time. Ceteris paribus, we would expect willingness to pay for an extra unit to decline with the number of units consumed. The per-unit price willingly paid when the consumption flow is \( q_i \) units is an average willingness to pay. To derive an inverse demand curve, we first must model the total value of the consumption stream, \( T_i = q_i Y_i \). (The threshold level of this variable must be transformed similarly: \( \tau_i = q_i \alpha_i \).) The implied marginal value of the last unit consumed is then the fitted value of the corresponding expression for \( dT_i/dq_i \).

To be consistent with our conviction that demand curves ought to slope downward, the \( dT_i/dq_i \) must be decreasing in \( q_i \). It is useful to digress briefly on potential specifications for \( T_i \). One possibility is the quadratic family.

\[ T_i = q_i Y_i = \beta_0 + \beta_1 q_i + \alpha_1 q_i^2 + \beta_2 x_i + \varepsilon_i \]
\[ dT_i/dq_i = \beta_1 + 2\alpha_1 q_i \]

Valuing Total Units of a Product

In formulating a model to address willingness to pay, the analyst must keep in mind whether the product being considered is purchased just once or repeatedly.

Valuing a Single Marginal Unit of a Product

In this case, technically we would be considering the inverse demand function for a single unit of a product with a particular configuration of characteristics (such as a large consumer durable good). In practice most analysts will be constrained by deficient data to working with plausible ad hoc specifications for the WTP function. In this context, reasonable first-generation models might include variants of linear \( \log(Y_i) = \beta_0 + \beta_1 q_i + \varepsilon \) or \( \log(Y_i) = \beta_0 + \beta_2 x_i + \varepsilon \) forms. (Whatever transformation is desired for \( Y_i \) is simply imposed on \( T_i \).)

Valuing Total Units of a Product

Alternatively, the proposed product might be a close substitute for a product that the consumer currently purchases regularly (such as public transportation, beer, long distance telephone services, or weekly newsmagazines).\textsuperscript{7} The survey should establish the number of units of the good typically purchased per unit of time. Ceteris paribus, we would expect willingness to pay for an extra unit to decline with the number of units consumed. The per-unit price willingly paid when the consumption flow is \( q_i \) units is an average willingness to pay. To derive an inverse demand curve, we first must model the total value of the consumption stream, \( T_i = q_i Y_i \). (The threshold level of this variable must be transformed similarly: \( \tau_i = q_i \alpha_i \).) The implied marginal value of the last unit consumed is then the fitted value of the corresponding expression for \( dT_i/dq_i \).

To be consistent with our conviction that demand curves ought to slope downward, the \( dT_i/dq_i \) must be decreasing in \( q_i \). It is useful to digress briefly on potential specifications for \( T_i \). One possibility is the quadratic family.

\[ T_i = q_i Y_i = \beta_0 + \beta_1 q_i + \alpha_1 q_i^2 + \beta_2 x_i + \varepsilon_i \]
\[ dT_i/dq_i = \beta_1 + 2\alpha_1 q_i \]

DEMONSTRATION OF AN EMPIRICAL IMPLEMENTATION

As we do not have available a sample from a market research application, we illustrate our procedures with a very simplified version of the model (and a subset of the data) used in a nonmarket resource valuation study (Cameron and James 1986, 1987), emphasizing the parallels to potential marketing studies.

For this illustration, our sample consists of 1033 responses to an in-person survey of recreational salmon anglers returning from fishing excursions. After assessing the actual level of incidental expenses for the fishing day, respondents were asked if they still would have gone fishing if the fishing day had cost some (randomly assigned) number of dollars more. As explanatory variables in this simple illustration, we use NFISH (the number of salmon caught), LGFISH (the weight of the largest fish in pounds), MEANT (mean temperature that day in degrees Celsius), TOTPREC (total precipitation in millimeters), and a dummy variable, NONRES (which takes a value of one if the respondent is not a local resident and is zero otherwise). In the context of the discussion in the preceding section, the "product" in question is a single fishing day; NFISH and LGFISH are characteristics of the fishing day (\( c_i \)), NONRES describes the respondent (\( z_i \)), and MEANT and TOTPREC might be considered analogous to the macroeconomic environment wherein purchase is being considered (\( w_i \)). Table 1 summarizes the data.

---

\textsuperscript{9}This specification will allow \( x \) to shift the marginal WTP curve in \((Y,q)\)-space.

\textsuperscript{10}This marginal WTP will be positive as long as \( \beta_i > 0 \) and downward sloping as long as \( (\beta_0 + \beta_1 \log q_i + \beta_2 x_i) > \beta_i \). The \( x \) vector will affect the position of this curve.

\textsuperscript{11}In our application to the valuation of a recreational fishery, we use Box-Cox transformations of the implicit dependent variable (Cameron and James 1986).
Table 2 gives the maximum likelihood estimates of $\beta$ and $\sigma$ (and their asymptotic t-test statistics) for the likelihood function in equation 8, assuming a very simple linear specification for the valuation function. Also shown for comparison are point estimates of $\beta$ and $\sigma$ computed from estimates of $\gamma^*$ from an ordinary probit model (and approximate asymptotic t-ratios using standard errors computed by the Taylor series approximation given in equation 10).

In the past, empirical investigators using contingent valuation data used inappropriate interpretations of their discrete choice models. As a consequence, they were limited to specific forms for their underlying WTP functions to ensure tractability in solving their probability function in equation 8, assuming a very simple linear specification for the valuation function. Also shown for comparison are point estimates of $\beta$ and $\sigma$ computed from estimates of $\gamma^*$ from an ordinary probit model (and approximate asymptotic t-ratios using standard errors computed by the Taylor series approximation given in equation 10).

In the past, empirical investigators using contingent valuation data used inappropriate interpretations of their discrete choice models. As a consequence, they were limited to specific forms for their underlying WTP functions to ensure tractability in solving their probability function in equation 8, assuming a very simple linear specification for the valuation function. Also shown for comparison are point estimates of $\beta$ and $\sigma$ computed from estimates of $\gamma^*$ from an ordinary probit model (and approximate asymptotic t-ratios using standard errors computed by the Taylor series approximation given in equation 10).

In the past, empirical investigators using contingent valuation data used inappropriate interpretations of their discrete choice models. As a consequence, they were limited to specific forms for their underlying WTP functions to ensure tractability in solving their probability function in equation 8, assuming a very simple linear specification for the valuation function. Also shown for comparison are point estimates of $\beta$ and $\sigma$ computed from estimates of $\gamma^*$ from an ordinary probit model (and approximate asymptotic t-ratios using standard errors computed by the Taylor series approximation given in equation 10).

In the past, empirical investigators using contingent valuation data used inappropriate interpretations of their discrete choice models. As a consequence, they were limited to specific forms for their underlying WTP functions to ensure tractability in solving their probability function in equation 8, assuming a very simple linear specification for the valuation function. Also shown for comparison are point estimates of $\beta$ and $\sigma$ computed from estimates of $\gamma^*$ from an ordinary probit model (and approximate asymptotic t-ratios using standard errors computed by the Taylor series approximation given in equation 10).

In the past, empirical investigators using contingent valuation data used inappropriate interpretations of their discrete choice models. As a consequence, they were limited to specific forms for their underlying WTP functions to ensure tractability in solving their probability function in equation 8, assuming a very simple linear specification for the valuation function. Also shown for comparison are point estimates of $\beta$ and $\sigma$ computed from estimates of $\gamma^*$ from an ordinary probit model (and approximate asymptotic t-ratios using standard errors computed by the Taylor series approximation given in equation 10).

The $\beta$s in this very simple specification give the change in WTP for a one-unit change in the level of each explanatory variable. As expected, both number and size of fish caught increase the respondent's WTP: an additional fish adds about $1.84 to WTP; if the largest fish is one pound heavier, WTP is higher by about $50. MEANT is correlated with sunshine. Because the fish descend to greater depths in bright sunshine and become more difficult to catch, it is not surprising that higher values of MEANT imply a less valuable fishing day.\(^{12}\) TOTPREC does not affect the fish, but definitely makes the angler less comfortable: for every millimeter of rain, WTP is decreased by about $1.02. Nonresidents have traveled farther to go fishing and hence can be expected to value the experience more—by $12.01, on average.

Because of the invariance property of maximum likelihood estimation, the point estimates of $\beta$ and $\sigma$ should be identical whether they are obtained directly by our new method or as transformations of the conventional maximum likelihood probit parameters. Also, in this case, the distortion in the standard error estimates due to use of the Taylor series approximation is relatively small. This fact suggests that the model can be estimated readily by current computer programs for conventional probit models. General nonlinear optimization programs are less common and typically require greater programming skill, so this result is very reassuring.

It is useful to compare our results with those from a probit analog of the Sellar, Stoll, and Chavas (1985) and Sellar, Chavas, and Stoll (1986) technique. We divide the $0$ to $100$ range of observed values into 10,000 one-cent intervals, then "integrate" the area under the two-dimensional $\Pr(I, = 1)$ curve (evaluated at the means of the other explanatory variables) across this range of values.\(^{13}\) Previously investigators computed the change in WTP for a one-unit change in the level of each explanatory variable by permuting the mean of each variable by one unit and recomputing the mean WTP. For com-

\(^{12}\)Another possibility is that only avid anglers are out on bad days.

\(^{13}\)As best we can infer, this is approximately the procedure used in all previous analyses of this type of CV data.

---

Table 2

<table>
<thead>
<tr>
<th>Variable</th>
<th>Maximum likelihood parameter estimates(^a)</th>
<th>Parameters computed from probit model estimates</th>
<th>Derivatives implied by numerical integration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>29.60 (11.03)</td>
<td>29.60(^a) (14.26)</td>
<td>1.755</td>
</tr>
<tr>
<td>NFISH</td>
<td>1.842 (3.220)</td>
<td>1.842 (3.276)</td>
<td>.4716</td>
</tr>
<tr>
<td>LGFISH</td>
<td>.4976 (3.592)</td>
<td>.4976 (3.671)</td>
<td>.4716</td>
</tr>
<tr>
<td>MEANT</td>
<td>-.7293 (-3.933)</td>
<td>-.7293 (-4.038)</td>
<td>-.6879</td>
</tr>
<tr>
<td>TOTPREC</td>
<td>-1.017 (-2.958)</td>
<td>-1.017 (-3.002)</td>
<td>-.9577</td>
</tr>
<tr>
<td>NONRES</td>
<td>12.01 (3.979)</td>
<td>12.01 (4.087)</td>
<td>11.68(^a)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>14.56 (17.43)</td>
<td>14.56 (17.43)</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Implicit dependent variable is unobservable valuation (WTP).

\(^b\)Asymptotic t-ratios from maximum likelihood estimation are in parentheses.

\(^c\)Because of the invariance property of maximum likelihood estimators, the point estimates will be identical by either method.

\(^d\)Approximate t-ratios constructed from Taylor series standard error approximations.

\(^e\)Change in mean WTP as NONRES goes from 0 to 1.
comparison, these implied derivatives are reported in the last column of Table 2. The derivatives obtained from the numerical integration method systematically underestimate the β estimates obtained from the maximum likelihood estimation. This distortion is probably due to the truncation of the range of integration at $0 and $100.

Once our optimization process has yielded the required parameter estimates and standard errors, an empirical investigator usually will be interested in determining the within-sample goodness of fit of the estimated model. Here, we can adapt some measures traditionally used with discrete choice models: individual prediction success and aggregate prediction success. The results are reported in Table 3.¹⁴

In comparison with conventional methods for estimating models using closed-ended contingent valuation data, the new approach has a clear advantage in its ability to determine (systematically and simply) the effect on expected WTP of changes in the level of each explanatory variable. Nevertheless, market researchers still might be interested in the marginal mean of the distribution of individual WTP values. Mean WTP across all respondents is $23.40. Our probit analog to the Sellar, Stoll, and Chavas technique yields a mean WTP of $23.73 (though the cumulative probability function reaches .5 at exactly $23.40). This discrepancy is due to the fact that the lower end of the range for the numerical integration is truncated at zero. The upper truncation at $100 has less influence in this case.¹⁵

Of course, the marginal mean in either case depends fundamentally on the distribution in the sample of the X = (C, Z, W) variables. If, for example, planned changes in product attributes affect the values of some of the X variables but not others, one must know the incremental contribution of each variable to valuation before one can simulate the effects of such a change on the marginal mean distribution of WTPs. An ability to do this rigorously makes our model superior to its predecessors. For example, just as the effects of changes in X on the fitted value of Y can be simulated in an OLS model, the same sorts of changes can be simulated for CV data. In our empirical illustration, it is very easy to simulate a change in a "product characteristic," NFISH, to determine the likely effect on each individual's expected willingness to pay.

**SUMMARY**

Extensive evaluation of CV methods in experimental settings suggests they are very reliable, despite some early concerns discussed at length by Cummings, Brookshire, and Schulze (1986) and Mitchell and Carson (1987). These survey instruments are becoming increasingly popular and show considerable promise for assessing the market potential of products that have not yet been developed (or are not yet being produced in quantities large enough for actual test marketing).

To implement empirically the procedures we describe, one would select a random sample of potential consumers and give each consumer a randomly assigned "product scenario." Across the selection of product scenarios, the investigator is free to vary not only the proposed price, but also the levels of all other product attributes. Each consumer's willingness or unwillingness to purchase the specific product at the designated price is recorded. If the experimental design includes variability in the levels of prices, product attributes, and consumer characteristics, the researcher will be able to use the statistical techniques we describe to calibrate the demand function, identifying its sensitivity to all of those factors.

We do not suggest that CV methods should displace conventional pre-test-market analysis procedures, but they certainly should be added to the array of analytical tools available to researchers during the pre-test-market phase of product development.

**APPENDIX**

Using the notation established in the text, we first define the following simplifying abbreviations (z denotes the standard normal random variable in this appendix).

\[ z_i = (x_i - \mu)/\sigma \quad \phi_i' = \phi(z_i) \]
\[ \Phi_i = \Phi(z_i) \quad S_i = x_{i0} \phi_i^2 \]
\[ R_i = x_{i0} \phi_i' \quad U_i = x_{i0} \phi_i^2 \]
\[ T_i = x_{i0} \phi_i' \quad W_i = z_i^2 \phi_i^2 \]
\[ V_i = z_i^2 \phi_i \quad \phi_i' = \phi'(z_i) = -z_i \phi(z_i) \]

The gradient vector for this model then is given by

\[ \delta \log L/\delta \beta_r = (1/\sigma) \sum [(I_r - (1 - \Phi_i))x_{r0}\phi_i/[\Phi_i(1 - \Phi_i)]] \]

\[ r = 1, \ldots, p \]
\[ \frac{\partial \log L}{\partial a} = (1/\sigma) \sum \left[ \frac{I_i (P_i - 1) Q_i}{\Phi_i (1 - \Phi_i)} \right]. \]

The elements of the Hessian matrix can be simplified if we define the function

\[ G(P, Q) = \sum \left[ \frac{I_i (P_i [\Phi_i - 1] - Q_i)}{\Phi_i^2} + \frac{(1 - I_i) (P_i [\Phi_i - Q_i])}{\Phi_i} \right]. \]

Then

\[ \frac{\partial^2 \log L}{\partial a \partial a} = (1/\sigma^2) G(R, S), \quad r, s = 1, \ldots, p \]
\[ \frac{\partial^2 \log L}{\partial a \partial a} = (-1/\sigma^3) \frac{\partial \log L}{\partial a} + (1/\sigma^2) G(T, U), \]
\[ r = 1, \ldots, p \]
\[ \frac{\partial^2 \log L}{\partial a \partial \phi_i} = (-1/\sigma) \frac{\partial \log L}{\partial a} + (1/\sigma^2) G(V, W). \]

Use of these analytic derivatives instead of numerical approximations can reduce computational costs considerably if one is estimating equation 8 directly.

Because the expectation of \( I_i \) is \( (1 - \Phi_i) \), the negatives of the expectations of these elements are:

\[ \mathbb{E}(\partial^2 \log L/\partial a \partial a) = (1/\sigma^2) \sum \frac{\phi_i^2 x_i y_i}{\Phi_i (1 - \Phi_i)} \]
\[ r, s = 1, \ldots, p \]
\[ \mathbb{E}(\partial^2 \log L/\partial a \partial \phi_i) = (1/\sigma^2) \sum \frac{\phi_i^2 y_i z_i}{\Phi_i (1 - \Phi_i)} \]
\[ r = 1, \ldots, p \]
\[ \mathbb{E}(\partial^2 \log L/\partial \phi_i \partial \phi_i) = (1/\sigma^2) \sum \frac{\phi_i^2 z_i^2}{\Phi_i (1 - \Phi_i)}. \]

REFERENCES


